SOCIETY FOR THE ENCOURAGEMENT OF
ARTS, MANUFACTURES & COMMERCE.

Cantor Lectures

ON

THE MEASUREMENT OF HIGH FREQUENCY CURRENTS AND ELECTRIC WAVES.

BY

J. A. FLEMING, M.A., D.Sc., F.R.S.
Professor of Electrical Engineering in University College, London.

Delivered before the Society of Arts on November 27, *and December* 4, 11, *and* 18, 1905.

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SYLLABUS.

Lecture I.

Measurement of High Frequency Capacity, Inductance, and Resistance.—Practical methods of measuring small capacities—Commutator and Bridge Methods—Capacity of Leyden jars, aërial wires, antennæ and insulated conductors—Formulæ for capacity of standard forms—Practical measurement of small inductances—Pre-determination of inductances of standard circuits—Formulæ for important cases—High frequency resistance—Surface distribution of high frequency currents—Effects of frequency on inductance and resistance—Methods of determining spark resistance—Methods of measuring number of sparks per second—Effect of proximity of aërial wires on their capacity and inductance.

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THE MEASUREMENT OF HIGH FREQUENCY CURRENTS AND ELECTRIC WAVES.

Lecture I.—Delivered November 27, 1905.

Measurement of High Frequency, Capacity, Inductance, and Resistance.

On two previous occasions, I have had the honour of giving lectures in this room "On Electric Oscillations" and "On Hertzian Wave Telegraphy." Since that time, our knowledge of the subject has made great progress, especially with regard to quantitative measurements. When any branch of scientific research begins to receive technical application, we always find that it calls for processes of exact measurement, and, in truth, we may say that any collection of facts only becomes scientific knowledge, just in proportion as it is capable of being made the subject of precise measurement.

In these lectures, I propose then to direct your attention to the "Measurement of High Frequency Electric Currents and Electric Waves," and the course may be in one sense considered as supplementary to, or a continuation of the two previous ones. Limitations of time will render it necessary for me to assume that you are familiar to a certain extent with the general phenomena.*

The first matter which must engage our attention is the measurement of electric capacity. When a charge of electricity is given to a conductor, the result is to raise what is called its potential, just as when a quantity of heat is imparted to a body, the result is to raise its temperature, or when a quantity of water is put into a vessel the result is to raise the level of the water. Potential is therefore to electricity what temperature is to heat and level to liquids. Heat tends to flow from places of high temperature to places of low. Water tends to flow down hill, and electricity to move in the direction in which potential diminishes most quickly.

* For further information, the reader is referred to the previous Courses of Cantor Lectures on "Electrical Oscillations and Electric Waves" (1900) and "Hertzian Wave Telegraphy" (1903).

The blocks illustrating these lectures have been kindly lent by the proprietors of *Engineering*.

The ratio of charge to potential is called the capacity of the body, and capacity may be defined as the electric charge required to raise a conductor to unit potential. The potential of the earth is taken as the zero. Accordingly we have the following definition:—

Definition.—The electrical capacity of a conductor is measured by the quantity of electricity which must be put into it to raise its potential one unit above that of the earth, all other conductors being at a great distance.

Since there is no absolute potential any more than absolute level, we take the earth's potential as zero. We have next to settle the units. We have first a definition of the unit of electric quantity.

Definition.—The electrostatic unit of electricity is the quantity which repels another equal quantity at a distance of one centimetre with a unit of force. The unit of force is called one *dyne* and is $\frac{1}{981}$ part of the weight of one gramme.

A conductor charged with electricity possesses a store of energy, and requires expenditure of work to charge it. It gives up this energy on discharge. Hence we have a further definition.

Definition.—A body is charged to unit potential when a unit of electric quantity on it represents a store of one unit of energy.

We may, therefore, employ a small conductor charged with one electrostatic unit of quantity to measure the potential of any other charge by ascertaining how much work is required to move the unit charge against the electric attraction or repulsion of the other charge to or from an infinite distance, or which comes to the same thing, to or from the earth. The law governing the electric attraction or repulsion between particles of electricity is the same as that of gravitation between particles of matter, viz., the law of the inverse square.

Imagine then a sphere of radius R, charged with Q units of electricity. It exercises an attraction of Q/d^2 units of force on a unit of charge of opposite sign placed at distance of

d centimetres from its centre, and it can be shown by a simple calculation that the work done in moving a charge to an infinite distance is Q/d units of energy, or ergs. Hence we have $V = Q/R$ or $C = Q/V = R$, and, therefore, that the capacity of a sphere of radius R centimetres is R electrostatic units.

The practical unit of capacity is called 1 microfarad, and is equal to 900,000 electrostatic units. The capacity of the whole earth is about 800 microfarads, or nearly the same as that of an Atlantic cable. The microfarad is, however, too large a unit for the measurements we have in view, and hence I employ the one-millionth part of it, or the micro-microfarad (m.mfd.), as a convenient unit. This is equal to that of a sphere 9 millimetres in radius, about the size of a large cherry, whereas the microfarad is the capacity of a sphere 9,000 metres, or about 5½ miles in radius. It is important to bear in mind the rule that to convert capacity measured in electrostatic units into capacity measuring microfarads we must divide by 900,000.

There are certain regular forms of conductors for which the electrostatic capacity can be mathematically calculated. These are the sphere, a circular disc, an ellipsoid, a long straight wire, and the following Table I. shows the mathematical expressions for these capacities in electrostatic units (E.S.) and in micro-microfarads (m.mfds.).

TABLE I.

Capacity of	In E.S. Units.	In m.mfds.
Sphere	R	$10\,R/9$
Disc	$2R/\pi$	$20\,R/9\pi$
Vertical Wire	$l/2 \log_\epsilon 2h/d$	$l/4 \log_{10} 2\,h/d$
Horizontal Wire	$l/2 \log_\epsilon 4h/d$	$l/4 \log_{10} 4\,h/d$
Parallel Plates	$A/4\pi t$	$10\,A/36\pi t$
Concentric Cylinders..	$l/2 \log_\epsilon \frac{D}{D'}$	$l/4{\cdot}15 \log_{10} \frac{D}{D'}$

The letters in the above formulæ signify as follows :—

l = length of conductor.
R = radius of sphere or disc.
d = diameter of wire.
h = height above the earth.
D = diameter of the sphere, disc, or cylinder.

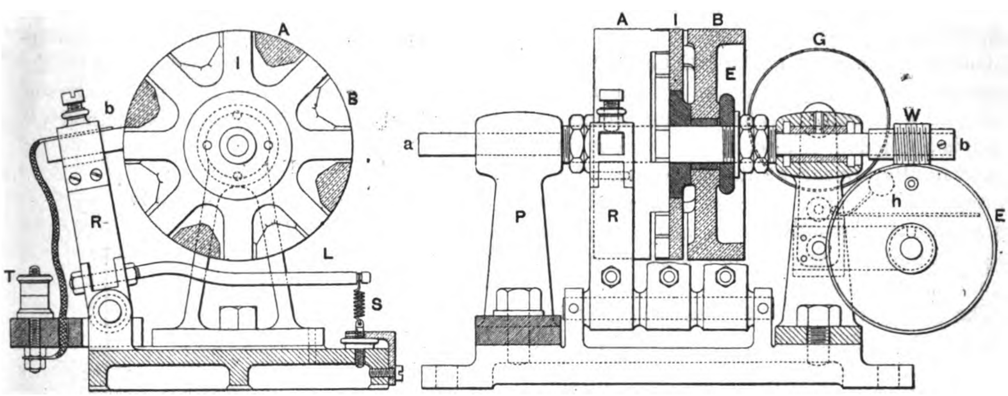
The proximity of any other conductor, even the earth, raises the capacity of the conductor. A large sphere hung up in this room would have a capacity numerically greater than its radius when reckoned in electrostatic units by reason of the proximity of the walls of the room.

The only method of finding the capacity of most insulated conductors is to do it experimentally. This is best achieved in the following manner :—If we charge the conductor and discharge it through a galvanometer, say, 100 times per second, these numerous discharges are equivalent in effect to a continuous electric current. Suppose that the galvanometer has been calibrated so that we can interpret its deflections and ascertain the value of the current in microamperes, which produces any given deflection. Let V be the potential to which the conductor is charged, C, its capacity in microfarads, and N, the number of discharges per second, we have then the relation, $A = C\,N\,V$. Hence the capacity can be determined when the numerical values of A, N, and V are known. This process is best conducted by means of a rotating commutator. The particular form of commutator here exhibited was designed by the author in conjunction with Mr. Clinton. (See Fig. 1.) It consists of a barrel or drum driven by an electric motor. This drum consists of three parts, the two outside portions have projecting teeth like those of a wheel, and there is an intermediate wheel which is insulated from and fixed between the other two. The diagram in Fig. 2 shows the development of the surface of this drum. Against the drum, three brass brushes press, two against the outer portions and one against the middle, and it will easily be seen that when the drum rotates the effect is alternately to connect the middle brush first with one and then the other of the two outside brushes. To conduct the measurement we employ a mirror galvanometer and an insulated battery composed of 50 or more small secondary cells. The conductor, C, the capacity of which is to be measured, is connected to the middle brush, and the two outside brushes are connected respectively to one terminal of the battery, B, and of the galvanometer, G, the other terminals of the battery and galvanometer being connected together and to the earth. (See Fig. 3.) On setting the motor in rotation the conductor is connected alternately with the battery and galvanometer, these being charged to a known potential and then discharged through the galvanometer. The number of times per second this operation is performed is ascertained in the following manner :—On the shaft of the motor is placed an endless screw gearing in a wheel which rotates once every 100

rotations of the motor. A pin on this wheel is caused once in every 100 revolutions to move the hammer of a small gong and thus to make a sound which announces that the motor has completed 100 revolutions.

In the commutator as constructed in the instrument before you, one revolution of the motor causes four discharges to pass through the galvanometer. Hence, if we take the time by the stop watch in which the motor makes

FIG. 1.

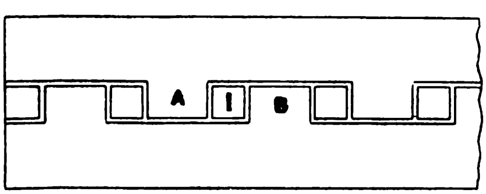


FLEMING AND CLINTON COMMUTATOR.

1,000 revolutions, or in which 4,000 discharges pass through the galvanometer, we can ascertain the number of discharges per second.

To reduce the results of observations to numerical values we have to know, first, the potential to which the conductor is charged. This may be done by measuring the difference of potential of the ends of the battery with any good form of voltmeter. The galvano-

FIG. 2.

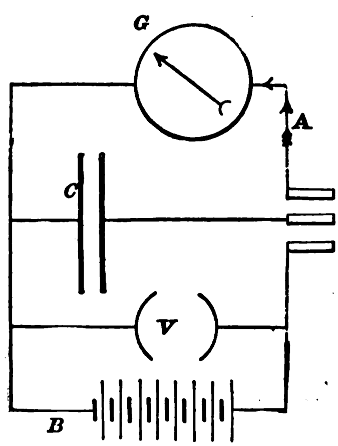


meter may be calibrated by passing through it a known current from the same battery obtained by connecting in series the galvanometer, the battery, and a known high resistance. Having these data and the number of discharges per second obtained as above described, we can measure the capacity of any insulated conductor, such as a large sphere, or disc, or vertical wire, and determinethe same in electrostatic units or micro-microfarads.

Again, by the same appliance, we can determine quite easily the capacity of a Leyden jar, or of a parallel plate condenser having between its plates a sheet of any solid dielectric. We can make use of the above-described commutator to measure the electrostatic capacity of an aerial wire, as used in wireless telegraphy, by simply connecting the insulated wire to the middle brush.

Such an insulated wire constitutes a form of Leyden jar, of which the earth is the outer

FIG. 3.



$A_{ma} = C_{mfd}\, VN$

coating, and the space round the wire the dielectric.

In making such a measurement, however, we have to determine the capacity of the commutator itself, and to subtract this from the

total capacity as measured. Operating with this appliance, we find the capacity of a Leyden jar, which is commonly called a pint size, may be about 1/700 of a microfarad whilst the capacity of a wireless telegraph antenna 100 feet in height, constructed of a wire 1/10 of an inch in diameter, may be about 1/5000 of a microfarad.

Experiments with the commutator show that if a number of wires are hung up parallel to one another in the air, the capacity of the whole of the wires taken together is much less than the sum of each of them taken individually when the others are removed.

Our attention must next be directed to the measurement of a quantity, called the inductance of a circuit. When an electromotive force is applied to a circuit, the current does not instantly appear at its full value. Time is required to create a current in a conductor by a finite electromotive force, just as time is required to create velocity in a mass of matter by mechanical force.

The quality of the circuit, however, on which this fact depends is called its inductance, and it corresponds to inertia in matter. The value of the inductance of a circuit can be calculated for certain simple cases, just as the capacity can be determined. If the wire is not made of magnetic material the inductance depends only on the size and shape of the wire, and is measured for our present purposes in centimetres or in larger units called microhenrys, each of which corresponds to 1,000 centimetres.

The mathematical processes by which these calculations can be made are somewhat complex, but in Table II. below are given the values of the inductance L for a number of cases with which we are concerned in practice, such as a straight wire of length l and diameter d, the same wire bent into a circle of perimeter l or into a square of perimeter l, or into a rectangle, the sides of which are respectively a and b.

TABLE II.

Inductance of Various Standard Forms of Circuit.

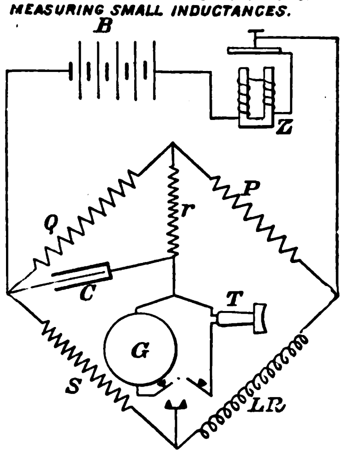
Straight wire ..	$= L = 2l\ 2{\cdot}303\ \log_{10} 4l/d - 1)$ cms.
Circular wire ..	$= L = 2l\ (2{\cdot}303 \log_{10} 4l/d - 2{\cdot}45)$
Square wire ..	$= L = 2l\ (2{\cdot}303 \log_{10} 4l/d - 2{\cdot}85)$
Two parallel wires	$= L = 2l\ (2{\cdot}303 \log 2D/d)$
Rectangle.. ..	$= L = 4\ [(a+b) \log_\epsilon \frac{4ab}{d} - a \log_\epsilon (a + \sqrt{a^2+b^2}) - b \log_\epsilon (b + \sqrt{a^2+b^2}) + 2\sqrt{a^2+b^2} - 2(a+b)]$

We have in the next place to notice a special characteristic to high frequency currents, viz., that the current is entirely on the surface of the conductor. An experimental proof of this is given in a later lecture.

The formulæ in the above Table II. give us the inductances for infinite frequency, or when the current is wholly upon the surface of the wire. They need a slight but unimportant correction when we are dealing with currents of finite frequency such as a million per second. In general, the only method of determining the inductance of a circuit is to do it experimentally. One of the most convenient means of carrying out this measurement is by a bridge measurement suggested by Professor Anderson and modified by myself. In this method we

FIG. 4.

THE ANDERSON FLEMING METHOD OF MEASURING SMALL INDUCTANCES.



$L_{cms} = 1000\ C_{mfds} \{r(R+S) + R\,Q\}$ Ohms.

make the conductor, the inductance of which is to be determined, one arm of a Wheatstone's bridge and complete the circuit as shown in Fig. 4. A variable resistance r is placed in series with the galvanometer and a condenser C connected between one terminal of the bridge and of the galvanometer. In the circuit of the battery is placed a buzzer or electromagnetic interrupter, Z, which breaks and makes the circuit of the battery very rapidly, say, from 200 to 300 times per second, and a telephone, T, is placed in the bridge circuit, which can be substituted for the galvanometer, G, when required. The first step is to balance the bridge for steady currents.

This is done by employing the galvanometer in the bridge circuit, and stopping the buzzer. When the measuring arm of the bridge has been so adjusted that the galvanometer shows

no current, the battery key being held down, we know that the relation which holds good between the four arms of the bridge, P, Q, R, and S is that P/Q = R/S. We then set the buzzer in operation, and substitute the telephone for the galvanometer, and alter the value of the resistance *r* in series with the galvanometer until the telephone yields no sound. If then C is the capacity in the bridge reckoned in microfarads the inductance L in centimetres of the conductor being tested is given by the formula :—

$$L = 1{,}000\ C \left\{ r\,\overline{R + S} + R\,Q \right\} \text{ centimetres,}$$

where C is the capacity of the condenser in microfarads, and P, Q, R, and S are the bridge arm resistances in ohms. L is the inductance of the arm of the bridge, of which the resistance is R.

In making this measurement of inductance, the capacity which must be employed in the bridge circuit has to be selected with reference to the magnitude of the inductance being measured. We may employ the method to determine the inductance of a wire bent into the form of a square, or, *vice versâ*, we may employ the known inductance of such a wire to determine the value of the capacity being used in connection with the bridge.

Another important measurement is that of the mutual inductance of two circuits. If one circuit acts inductively on another the mutual inductance M of the two circuits is measured by the electromotive force produced in the secondary circuit when a current of unit strength in the primary circuit is started or stopped uniformly in one second. If a current in a wire is steady, the fall of electric pressure, otherwise called the voltage, dropped down the wire is measured by the product of the resistance and current. If the current is rapidly varying the voltage drop is measured by the product of the inductance and the rate of change of the current. If we are dealing with alternating currents which have a uniform frequency, *n*, then the voltage drop due to inductance is easily shown to be equal to 6·24 times the frequency multiplied by the product of the current and the inductance. Hence if the frequency is very large, the voltage drop may be large, even if the current is small. It is easy to show this by a striking experiment.

We create an oscillatory discharge in a thick wire by sending Leyden jar discharges through it, and we find that if a small incandescent lamp is connected to two points on this wire a few inches apart, the lamp will light up, although the actual current passing, as shown on a hot wire ammeter, may not exceed 5 or 6 amperes. If, however, we pass a continuous current of the same value through the wire, we should find that the lamp would not light up. This shows that the volt drop down a given wire produced by a high frequency alternating current may be enormously greater than that produced by a continuous current of the same effective value.

Returning then to the mutual inductance, M, of two circuits, we may define this quantity by saying that the electromotive force produced in the secondary circuit by an alternating primary current is 6·28 times the frequency multiplied by the product of the primary current and the mutual inductance, M. If we have two coils of inductance, L and N, which form primary and secondary circuits of a transformer we can join them up in two ways; first, so that a current flows the same way round both coils, or secondly, we may join them up so that a current may flow in opposite currents round the two coils when these are joined in series.

It is easy to show that, in the first case, the whole inductance of the two circuits is L + 2 M + N, and in the second case, it is L − 2 M + N. Hence the difference of these two inductances divided by 4, gives us the value of the mutual inductance. We can, therefore, determine the mutual inductance of two circuits by making two measurements with the above described bridge and telephone, and can thus easily measure small mutual inductances.

It was mentioned just now that the principle property of high frequency currents is that they are entirely on the surface of conductors. A wooden rod, covered with the thinnest coating of gold leaf, is just as good a conductor for very high frequency currents as a solid copper one. On the other hand, the copper rod has a far higher resistance for high frequency currents than it has for steady currents. Lord Rayleigh has given a well known formula for tabulating the high frequency resistance, R′, of a nearly straight conductor of length *l*, of which the steady resistance is R ohms, for high frequency currents of frequency *n*. If the metal is non-magnetic, the formula is—

$$R' = \sqrt{R\,\pi\,n\,l}$$

and for a round-sectioned copper wire of diameter *d* the above formula becomes

$$R' = R \frac{\pi d}{80} \sqrt{n}$$

where R′ is the high frequency resistance, R the ordinary steady resistance, and l is the length of the conductor. We find, then, that for high frequencies, the value of R′ is very different from R. Thus, for instance, with a frequency 10^6 and a diameter of 1 centimetre, R′ is nearly 40 R. On the other hand, if $d = 1/40$ centimetre, which is the size of a No. 36 S.W.G., then for the same frequency R′ = R.

It is important to notice that the above formula only applies to straight or nearly straight wires, but not to closely-wound spirals, because, in the latter case, there is a displacement of the cylindrical distribution of the current over the cross section, which results in the production of eddy currents in the metal.

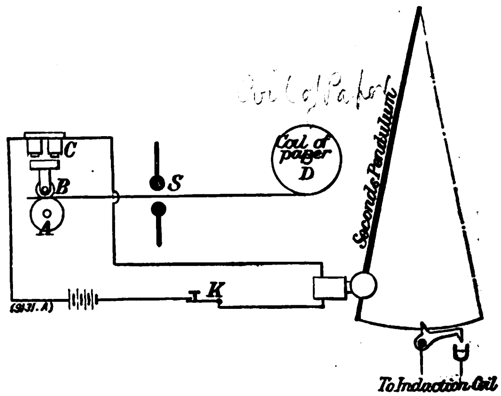
Accordingly, in making high-frequency circuits, all the conductors should be made of very thin strips of metal or of fine silk-covered wires bunched together, and then the value of the high-frequency resistance is known, and is equal to that of the ordinary or steady resistance.

Another measurement of considerable importance is that of a spark resistance. The usual mode of producing a high-frequeucy current, as explained in my previous courses of lectures, is by the oscillatory discharge of a condenser. To the secondary terminals of the induction coil we attach one terminal of a condenser, which may consist of a battery of Leyden jars ; the one coating of this condenser is attached to one terminal of the induction coil, and the other coating is attached to the other spark ball through an inductance coil. We have thus in series a spark gap, a condenser, and an inductance coil, and when the latter is put into operation, we have an oscillatory spark at the spárk gap, and trains of electric oscillations in the inductance coil. If we insert in series with this condenser an ammeter suitable for measuring high frequency currents, as described in the next lecture, we can by this means determine the effective value of the current in the oscillatory circuit. Again, if we know the length of the spark gap, we can determine the potential to which the condenser is charged, and if we know the capacity of this condenser we know the energy stored up in it.

If then we can determine the number of sparks per second, we can tell the power in watts given to the condenser, because this latter is equal to $\frac{1}{2} N \frac{C}{10^6} V^2$ where N is the number of discharges per second, C is the capacity in microfarads, and V the charging potential in volts, the resulting power being given in watts.

Suppose we assume that the inductance coil has no sensible resistance, then the whole resistance of the circuit is concentrated in the spark gap, and its resistance in ohms is given by the quotient of the power put into the condenser by the square of the current in amperes flowing in the condenser circuit. The measurement of the spark gap resistance therefore can be effected if we can determine the number of oscillatory sparks per second.

Fig. 5.



Spark Counter. (Fleming).

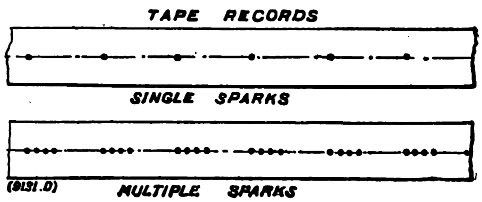
I have lately devoted particular attention to the construction of an instrument intended to effect this measurement. The instrument is constructed in the following manner :—A small electric motor has a flanged pulley put upon its shaft over which lies loosely a strip of telegraph paper tape, the pulley being just wide enough to allow the paper to lie on it. Over the pulley of the motor rests another pulley, called a jockey pulley, carried at the end of a long, weighted rod, which is upheld by an electromagnet. As long as the jockey pulley is held up, the paper tape will not travel when the motor revolves, but when the jockey pulley is dropped the paper is gripped and travels with a speed determined by the speed of the motor. This paper tape is made to pass in part of its course between two spark balls, and at every oscillatory spark it is pierced with a hole. The primary circuit of the induction coil contains a switch which can be closed by an electromagnet. This electromagnet is operated by a pendulum (see Fig. 5)

so arranged that when the pendulum takes one swing, occupying a period of one second, it breaks an electric circuit and then makes it again for exactly one second, and this interruption is caused to drop the jockey pulley on to the motor pulley for one second, and at the same time to close the induction coil circuit also for one second.

When the operation is over we find, therefore, that the paper tape is perforated with a number of holes which correspond to the number of sparks per second, that is, to the number of oscillatory discharges (see Fig. 6). Accordingly we have the means of reckoning the value of the letter N in the above formula.

We have already seen how the capacity of the Leyden jar can be measured, and in the next lecture figures will be given to you showing the manner in which the charging voltage can be estimated from the length of the spark gap. By means of this spark counter we are

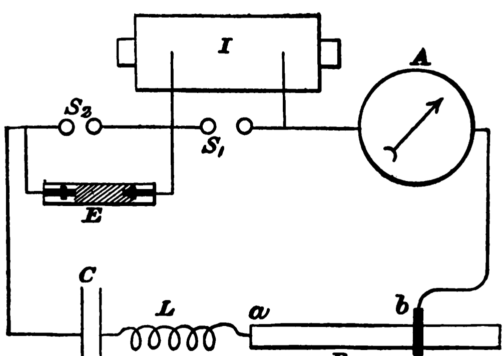
FIG. 6.



enabled to estimate the power put into the condenser, and from the measured value of the current in the oscillatory circuit we can determine the spark gap resistance. This spark gap resistance is found to be a function of many quantities. It depends first upon the length of the spark, increasing very rapidly with the length. In the second place, it varies with the quantity of electricity stored in the condenser and released at each discharge, the spark gap resistance decreasing with increasing discharge current. In the next place, it depends somewhat on the inductance of the circuit, and upon the frequency of the sparks, moreover a careful examination of the subject of the spark gap resistance, shows that the resistance of the spark is not constant over the whole operation of the spark, but increases towards the end of the discharge as the oscillations die away in amplitude. We cannot therefore speak of any particular length of spark as having a fixed resistance. A spark of 5 milliamperes in length may have a resistance of 5 or 10 ohms or it may have a resistance of very much less depending upon the quantity of the discharge.

Professor Slaby, of Berlin, has made some interesting experiments on spark resistance by the following method:—He places in series with one another two spark gaps, S_1 and S_2 (see Fig. 7). Across one spark gap are joined the secondary terminals of an induction coil and the other spark gap is short circuited by a tube, E, containing a solution of sulphate of copper, which has a resistance of about 410 ohms. The circuit is completed by a condenser, C, a variable inductance L, and a hot wire ammeter A, suitable for measuring high frequency currents and also a variable carbon resistance R. An experiment was conducted in the following manner:—The spark balls across which the induction coil terminals were connected will be called the main spark, the other spark gaps shunted by the tube of sulphate of copper will be called the spark gap under investigation. This last spark gap could have its length

FIG 7.



Slaby's Method of Measuring Spark Resistance.

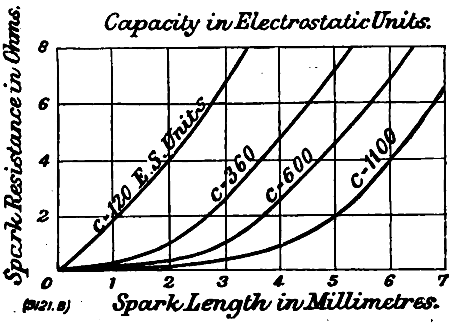
varied and measured by means of a micrometer. The first measurement consisted in closing this last spark gap entirely, and then varying the amount of resistance inserted by the carbon rheostat, whilst at the same time observing the value of the current as read on the hot wire ammeter. In this manner a curve could be plotted showing the variation of current with resistance in the oscillatory cireuit. In the next place the carbon rheostat was cut out and the length of the spark gap under observation was varied, whilst at the same time the readings of the hot wire ammeter were taken corresponding to observed spark gaps. From these two observations a curve could be set out showing the spark resistance in terms of the spark length.

Some of these observations by Professor Slaby are given in the following table, and plotted out on the curves in Fig. 8:—

TABLE III.—MEASUREMENT OF SPARK RESISTANCES. (Slaby.)

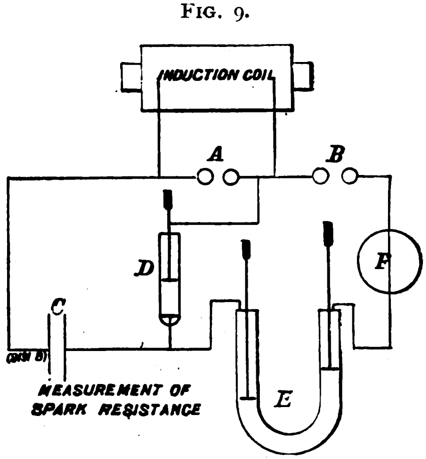
Spark length in m.m.	Spark Resistance in Ohms for various Capacities C in Electrostatic Units in the Circuit.			
	C = 120	C = 360	C = 600	C = 1,100
1 m.m.	2 ohms	0·4	—	—
2 ,,	4 ,,	1·0	0·25	—
3 ,,	7 ,,	2·7	1·0	0·4
4 ,,	—	5·0	2·5	0·8
5 ,,	—	7·5	5·0	2·0
6 ,,	—	—	7·2	4·2
7 ,,	—	—	—	6·5

FIG. 8.



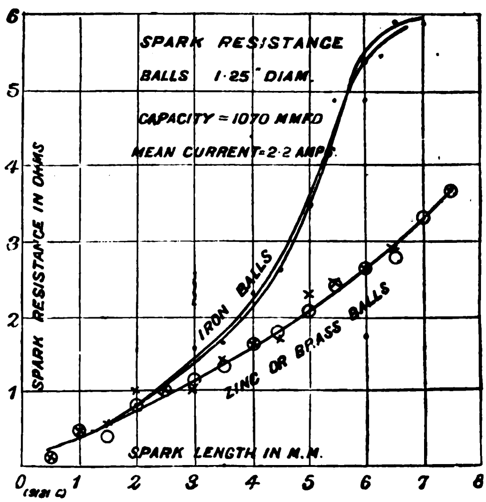
I have repeated Slaby's observations in the Pender Electrical Laboratory of University College, London, with some slight modifications. In place of the carbon rheostat

FIG. 9.



which I found liable to heat, I used a U-tube containing dilute sulphuric acid, having in it two moveable pistons, by means of which the length of the column of sulphuric acid in it could be varied (see Fig. 9). A thermometer, placed in this tube, served to give the temperature, and from the known resistivity it was possible to estimate the resistance for any position of the pistons. In place of short circuiting the spark gap by a tube of sulphate of copper I employed an inductance coil, and so arranged the ammeter that it measured only the current passing through the spark to be measured. These experiments were made with spark balls of different materials, and it was found that for lengths of spark gap, greater than two millimetres, there was a very decided difference between the resistance of sparks taken between iron balls and those between brass or zinc balls of about three centimetres in diameter (see Fig. 10). For spark lengths of about five or six millimetres the resistances of spark

FIG. 10.



between iron balls is nearly double that between zinc balls of the same size; this is no doubt connected with the greater infusibility of iron.

Further experiments on spark gap resistance have recently been made by Rempp. He employed rather long spark gaps, and took a spark between zinc balls varying in diameter from one and a-half to five centimetres. His results show that the spark resistance has a minimum value corresponding to a spark length of about four or five millimetres, the spark resistance increasing for shorter spark lengths, and also for longer spark lengths (see Figs. 11 and 12). Rempp confirmed the observations of Slaby and also my own, which show that in proportion as the capacity in the oscillating circuit is greater, so the spark resistance for a given spark length is smaller. The smallest spark length by Rempp was about half an ohm

when using a capacity somewhere near 7,000 m.mfds.

By means of the spark counter above described, I have made some intersting researches lately on the phenomena of the spark when using an alternating current transformer instead of an induction coil. Employing a transformer worked with an alternator having a frequency of about 50, I find that so far from there

FIG. 11.

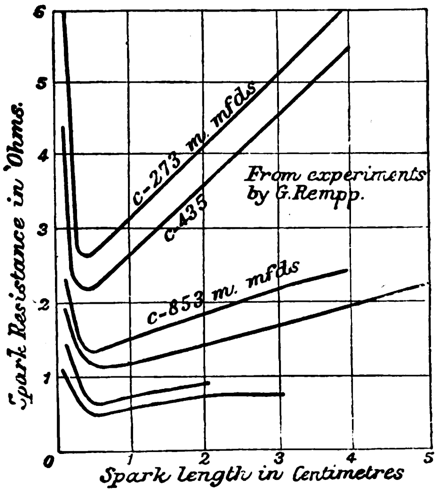
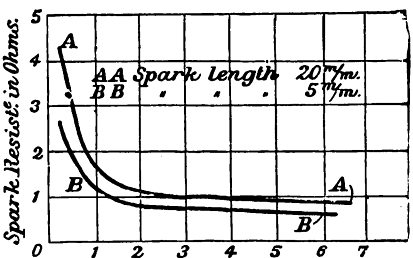


FIG. 12.

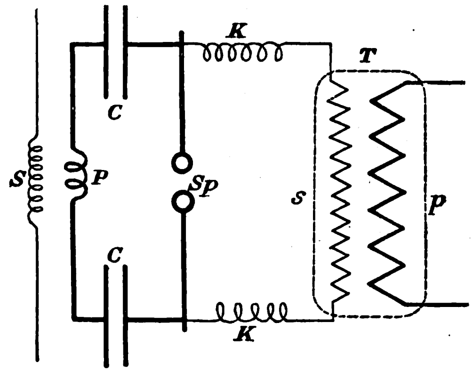


(9121.D)

being a spark for alternation, as might naturally be supposed, in some cases there were far fewer than 50 sparks per second, and in some cases a far greater number. When employing very short spark gaps, I found it was necessary to insert inductance in between the spark gap and the transformer, in order to destroy the alternating current arc that would otherwise form and enable the condenser to become charged (see Fig. 13). Under these circumstances, it is found that the oscillatory discharges sometimes rose to as many as 500 per second.

The explanation of this anomaly is as follows :—When an alternating current transformer is employed to charge a condenser placed in series with an inductance coil as a shunt across a spark gap, the interval between the spark balls is occupied not only by a condenser discharge, but by a true alternating current arc, and unless this arc is quenched at each alternation, the condenser cannot become charged again. The simplest way to quench this arc is to insert inductance between the spark balls and the transformer. This inductance, however, causes a drop in voltage when a current is taken out of the transformer. Consider then the instant before discharge. The transformer gives a certain

FIG. 13.



(9121.A) *Transformer Plant for creating Electric Oscillations*

voltage, the condenser is charged to that voltage, and if that voltage exceeds the voltage corresponding to the spark gap length, a discharge commences. At this moment, however, the voltage drops in consequence of the inductance inserted between the transformer and spark balls. The discharge then ceases, but it immediately commences again, and in this manner five or six discharges in the same direction may take place during one semi-period of the transformer current. On the other hand, if the condenser capacity is very large, and if the spark gap is long, it may require a time equal to several complete periods of the transformer current before the potential difference of the spark balls rises to a value which will cause a discharge to take place. It, therefore, follows that great errors may be committed if, in estimating the power put into the condenser, we always assume that the number of periods of the condenser is equal to the number of alternations in the alternating current; it may be much fewer and it may be much greater with corresponding difference in the resistance of the spark.

Lecture II.—Delivered December 4, 1905.

Measurement of High Frequency Current and Voltage.

If a condenser of any kind, such as a Leyden jar, is discharged through a low resistance, it is a familiar fact that the discharge does not consist in a single movement of electricity in one direction, but in a rapidly decreasing to-and-fro movement, called a train of electrical oscillatons. We can have this operation represented to the eye optically or photographically by means of a Duddell oscillograph, provided that we employ a capacity and inductance which are not too small.

The oscillograph is a kind of mirror galvanometer adapted to measure very rapidly changing currents. It consists of one or two loops of fine wire which carry a mirror, these loops being placed in a strong magnetic field (see Fig. 14). When an

Fig. 14.

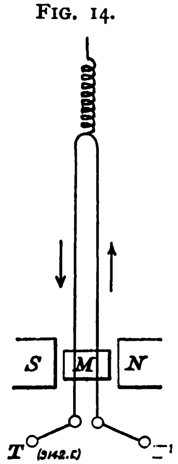
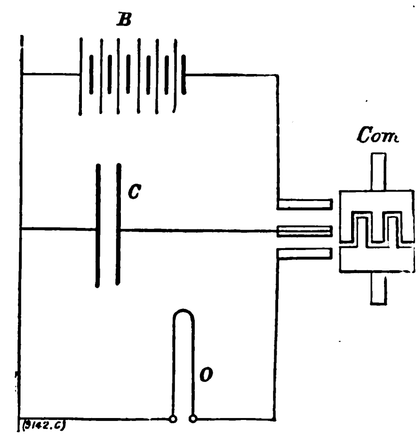


Diagram of Duddell Oscillograph.

alternating current is passed through the wire it causes the mirror to vibrate to and fro, and a ray of light reflected from this mirror is also reflected from another mirror with axis at right angles, which is vibrating in tune with the first. The ray of light is then reflected either on to a ground-glass screen or a sensitive photographic plate. In order to render the instrument available for recording non-periodic phenomena, I have adopted the following plan :—On the shaft of an alternator having a frequency of about 80, is fixed a disc of non-conducting material having brass sectors let into it. Against this disc three brass wire brushes press, in such a fashion, that, as the alternator revolves, the middle brush is alternately connected, first to one and then to the other of the outside brushes for a time, equal to half a period of the alternator. If a condenser C, a battery of secondary cells B, and the oscillograph circuit O are connected up, as shown in diagram in Fig. 15, we can then employ the current

Fig. 15.



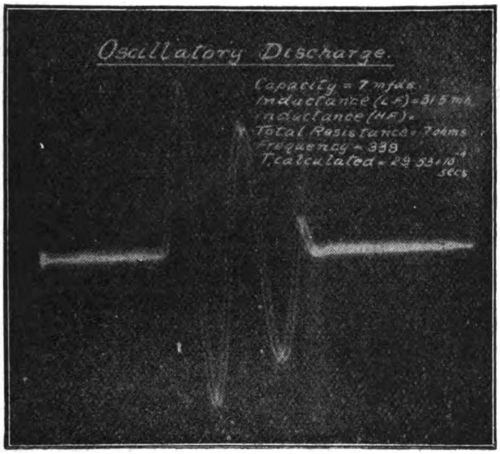
Arrangement for Delineating Condenser Discharge Curve.

from the alternator to drive the mirror of the oscillograph, and at the same time use the alternator shaft to drive the commutator which alternately charges the condenser from the battery and discharges it through the oscillograph.* The result of this is that we get on the screen of the oscillograph curves representing the oscillatory discharge of the condenser as seen in Figs. 16-20. These curves indicate that when the condenser discharges, the electricity rushes to and fro in the discharge circuit with a gradually decreasing amplitude, the oscillations decreasing in geometrical progression as the time increases in arithnetical progression. The capacity, inductance, and resistance in the circuit used are marked in each case on the diagram. The time interval between two successive zero points is called

* In the actual experiments made in the Pender Electrical Laboratory, the time during which the condenser was in connection with the oscillograph was $\frac{66}{10000}$ of a second, and the times of charge and discharge were not quite equal.

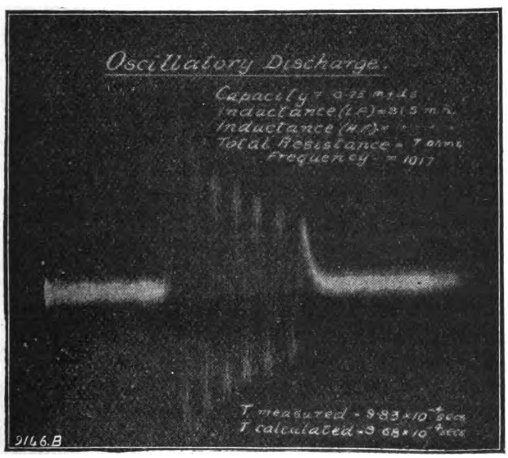
the semi-period of the oscillation, and the complete period is denoted by the letter T. The frequency, denoted by the letter n, is the number of periods per second. It can be shown by analysing these curves that two successive maxima in opposite

FIG. 16.



directions bear the same ratio to one another. In other words, if we can call I_1 I_2 I_3, &c., the successive maximum values of the currents, the ratio of I_1/I_2, I_2/I_3, &c., is constant. The logarithm of this ratio taken to the

FIG. 17.



Napierian base is called the logarithmic decrement, and denoted by the letter δ. If the amplitude of the oscillations dies away quickly, the oscillation is said to be highly damped, and if very slowly they are said to be slightly damped. If R is the total resistance of the circuit including the oscillograph, and L is the inductance, the ratio R/2L is called the damping factor, and denoted by the letter α. If C is the capacity of the condenser reckoned in microfarads, and L the inductance of the circuit in centimetres, then the quantity $\sqrt{CL}$ is called the oscillation constant of the circuit. It can be then shown that these quantities are connected by the following equations.—

$$T = \frac{1}{n} \quad p = 2\pi n = 6{\cdot}28n$$

$$\delta = \log_\epsilon \frac{I_1}{I_2} = \log_\epsilon \frac{I_2}{I_3} = \text{etc.}$$

$$\epsilon^\delta = \frac{I_1}{I_2} = \frac{I_2}{I_3} = \text{etc., where } \epsilon = 2.718$$

$$\alpha = R/2L = 2n\delta$$

$$T = \frac{2\pi}{\sqrt{\frac{1}{CL} - \alpha^2}} \text{ or } n = \frac{1}{2\pi\sqrt{CL}}$$

In making quantative measurements we must measure capacity in microfarads and inductance in centimetres, and then the formula for the frequency becomes—

$$n = \frac{5 \times 10^6}{\sqrt{C \text{ (in mfds)} \times L \text{ (in cms)}}}$$

and the expression for the oscillation constant O becomes—

$$O = \sqrt{CL} = \frac{5 \times 10^6}{n}$$

Also it can be shown that there is a certain relation between the first maximum ordinate I_1 of current, and the capacity of the condenser C the voltage V, to which it is charged and the frequency, which is expressed by the following formula—

$$I_1 = \frac{C}{10^6} V p$$

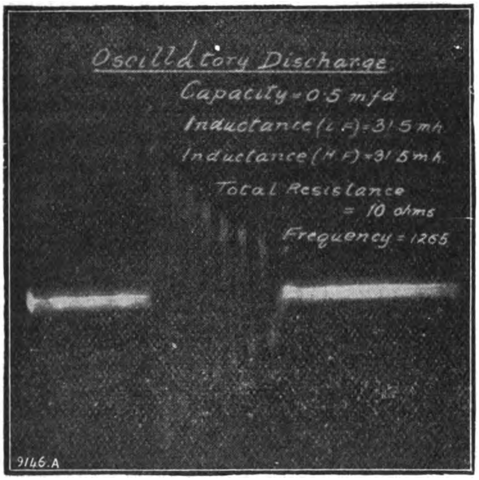
Where C is the capacity of the condenser in microfarads, V the voltage of the battery charging the condenser, and I_1 the first maximum value of the discharge current in amperes. The three important quantities in which we are concerned in considering the oscillatory discharge of the condenser are first, the frequency, n; secondly, the decrement, δ; and thirdly, the maximum amplitude, I_1. Again, we have to fix conventionally some limit beyond which we may consider the oscillations to be extinguished.

For all practical purposes we may consider the oscillations to be over when they are reduced to 1 per cent. of their initial value, It is then easy to show that the number of com-

plete oscillations, M, in the group or train is given by the formula—

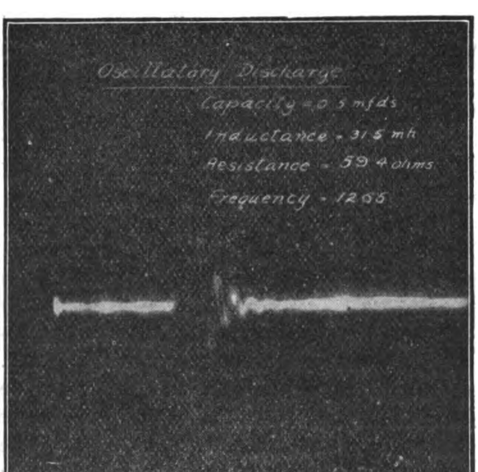
$$M = \frac{4 \cdot 606 + \delta}{2\delta}$$

FIG. 18.



Suppose δ has any such a value of 0·1, which means that each oscillation is about 90 per cent. of the preceding one. This means that in 23 or 24 oscillations, the discharge is finished. If, however, δ equals ·01, then there

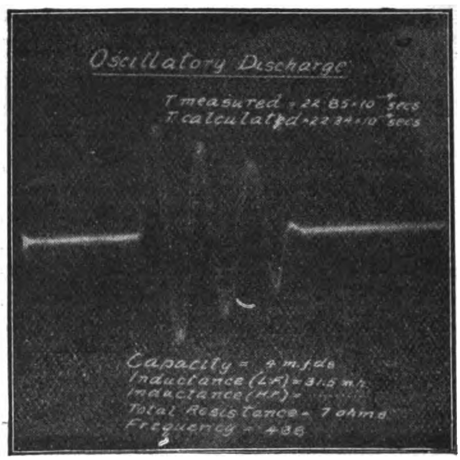
FIG. 19.



will be 230 to 240 complete oscillations in the train. In the case of oscillatory circuits as used in wireless telegraphy, in some cases there may not be half-a-dozen oscillations, in other cases as many as a thousand. As the operations of wireless telegraphy are greatly dependent upon the number of oscillations in a train, the measurement of the decrement is very important. We are now in a position to analyse completely the events which take place when a Leyden jar is attached to the secondary terminals of an induction coil, and has a spark gap and inductance placed in series, with it forming an oscillatory circuit with the spark gap.

We are concerned with the following five quantities: first, the capacity, C, of the jar or condenser; second, the inductance, L, of the circuit; third, the maximum voltage, V, to which the jar is charged; fourth, the number of discharges per second, N; and fifth, with the mean square value of the current in the oscillatory circuit, J^2. In order to explain

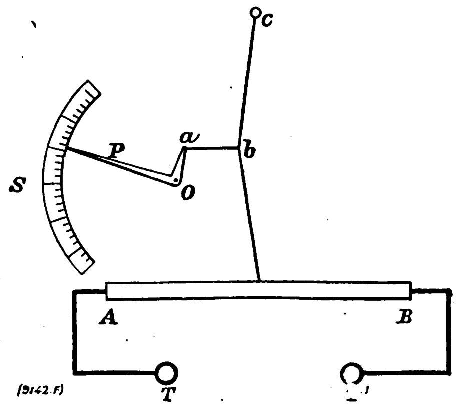
FIG. 20.



the meaning of this last term, consider the oscillations in one single train to be sent through a fine wire, they would create a certain amount of heat in that wire, and if N groups of oscillations were sent through the wire in one second they would create N times as much heat in the wire. Supposing then that we pass through the wire continuous current having a certain value J, such that it would produce in that wire in one second the same quantity of heat as the N groups of the oscillations. The current J is called the root-mean-square value of oscillations. We can therefore find the value of J or the root-mean-square value of any train of oscillations by passing it through a hot wire ammeter provided this has a suitable form. In order that such an instrument may be available for making the above measurement, the wire which is heated must consist of either one fine wire not exceeding No. 36 in size or else a group of such wires in

parallel, any suitable arrangement being made for measuring the expansion of the wire by heat, and therefore for graduating the instrument by means of a continuous current. I exhibit to you an instrument designed on these lines which I have had made. (See Fig. 21.)

FIG. 21.



HOT WIRE HIGH FREQUENCY AMMETER.

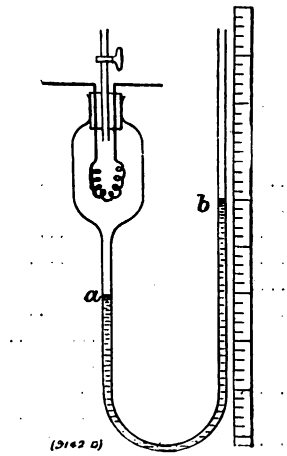
German investigators have made a good deal of use of a form of hot wire ammeter which is described in old books of electricity under the name of the Riess Electrical Thermometer. This instrument was, in fact, invented by our countryman, Sir W. Snow Harris, and was described by him in 1827 in the Philosophical Transactions of the Royal Society. In this instrument a fine wire or strand of fine wires is included in the bulb of an air thermometer consisting of a U-tube attached to a bulb, the bend of the tube being filled with some liquid. (See Fig. 22.) When a current is sent through the wire it heats the air and forces the liquid up in one leg of the U-tube until a stationary position is reached depending on the current. Such an instrument may be used for measuring the root-mean-square value of a train of oscillations, but my own experience is that it is not so useful or quick in operation as a hot wire ammeter made with a bundle of fine platinoid wires, the expansion of these wires being measured by means of the sag produced in them.

It is not difficult to show that there is a definite relation between the root-mean-square value of a train of oscillations, the maximum value I of the first oscillation, the logarithmic, decrement δ, and the frequency n, which is expressed by the following formula :—

$$J^2 = \frac{NI_1^2}{4a} = \frac{NI_1^2L}{2R} = \frac{NI_1^2}{8n\delta}$$

Returning then to the Leyden jar discharge, we place a hot wire ammeter in the discharge circuit and also a spark counter, and make use of a discharger, consisting of two spark

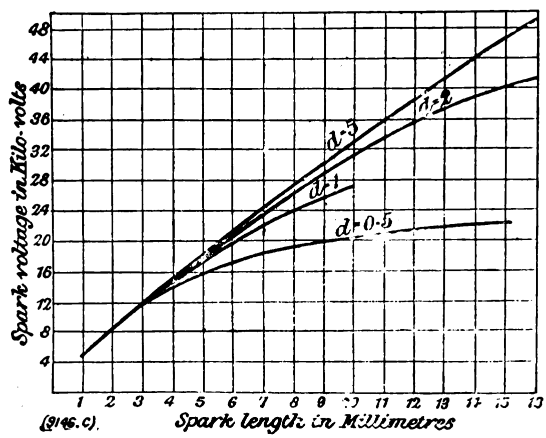
FIG. 22.



REISS OR SNOW HARRIS HOT WIRE AMMETER.

balls of steel, each ball being about 2 centimetres in diameter, the distance between the balls being adjustable and measurable by means of a micrometer screw. There is a definite relation between the voltage required

FIG. 23.



SPARK VOLTAGES FOR VARIOUS SPARK LENGTHS TAKEN WITH SPARK BALLS OF DIFFERENT DIAMETERS d IN CMS.

to give a spark of any length in air at the normal pressure, and the size of the spark balls, which is given in Table IV. below, mostly taken from Heydweiller's observations of spark potential.* (See Fig. 23.)

* See A. Heydweiller, "On Spark Potentials." *Ann. der Physik*, Vol. 48, p. 235 (1898).

TABLE IV.

SHOWING THE SPARK VOLTAGE BETWEEN BRASS BALLS, 2 CENTIMETRES IN DIAMETER, FOR VARIOUS SPARK LENGTHS.

Spark length in cms.	Spark voltage.	Spark length in cms.	Spark voltage.
0·1	4,700	2·7	54,900
0·2	8,100	2·8	55,800
0·3	11,400	2·9	56,700
0·4	14,500	3·0	57,500
0·5	17,500	3·1	58·300
0·6	20,400	3·2	59,000
0·7	23,250	3·3	59,700
0·8	26,100	3·4	60,400
0·9	28,800	3·5	61,100
1·0	31,300	3·6	61,800
1·1	33,300	3·7	62,400
1·2	35,500	3·8	63,000
1·3	37,200	3·9	63,600
1·4	38,700	4·0	64,200
1·5	40,300	4·1	64,800
1·6	41,300	4·2	64,500
1·7	43,200	4·3	66,000
1·8	44,700	4·4	66,600
1·9	46,100	4·5	67,200
2·0	47,400	4·6	67,800
2·1	48,600	4·7	68,300
2·2	49,800	4·8	68,800
2·3	51,000	4·9	69,300
2·4	52,000	5·0	69,800
2·5	53,000	5·1	70,300
2·6	54,000		

Supposing, then, that we connect the secondary terminals of an induction coil to the spark balls, and also connect the spark balls by means of an oscillatory circuit consisting of an inductance coil, a condenser or Leyden jar, and an ammeter suitable for measuring high frequency currents made as described; also let us suppose that a strip of paper tape is drawn between the spark balls, as already described, so as to count the number of sparks per second, and let it be assumed also that we have already measured, as described, the capacity of the Leyden jar and the inductance of the circuit, we have then the means for measuring the above five mentioned quantities, C, L, V, J, R, and from these measured quantities we can deduce the value of others, viz., the frequency, *n*, the decrement δ, the resistance of the circuit, R, in ohms, and the number of oscillations per train, N, by the following formula :—

$$I_1 = \frac{C}{10^6} Vp = V \frac{\sqrt{C/10^6}}{\sqrt{L/10^9}} = V \sqrt{\frac{1{,}000\ C}{L}}$$

$$R = \frac{N L I_1^2}{2 \times 10^9 J^2} \qquad \delta = \frac{R\ 10^9}{4 n L}$$

$$M = \frac{4{\cdot}606 + \delta}{2\delta}$$

Where C the capacity is meaured in microfarads, V the charging voltage in volts, L the inductance in centimetres, J the root-mean-square current in amperes, and the resistance R is given in ohms.

Let us apply this to the case of the Leyden jar before us. The jar has a capacity of $\frac{1}{400}$th mfd, hence C = 1/400 mfd., the inductance L of the circuit = 2,000 cms., the spark length = about 3 m.m., hence the voltage V = 12,000 volts, the number of sparks per second N = 50, and the R.M.S. value of the discharge current = J = 1·5 amperes. We then find that the frequency $n = 2{\cdot}25 \times 10^6$ and the maximum current I = 420 amperes. The spark resistance R = 3·5 ohms and the decrement δ = 0·2 and the number of oscillations per train M = 12.

Nothing is more astonishing than the large currents which can be given out by condensers of quite moderate size.

At one instant during its discharge this Leyden jar before you is furnishing a current of nearly 400 amperes, but of course that current lasts only for an infinitely small time. Supposing we desire to make a diagram to a scale representing what is taking place during one second when this Leyden jar is creating an oscillatory discharge, let us suppose that we take a scale of time on which one foot represents one millionth of a second, then a time of one second would be represented by a length of 200 miles. Again, let us suppose that we take a scale of current on which one millimetre represents one ampere, then we desire to represent a curve delineating the oscillations taking place in one second when this jar is discharged, we should have to draw a series of diagrams, each of which will resemble the one in Fig. 19, which would have to be placed four miles apart. Each train of twenty-four oscillations would be 4 feet 6 inches long, and the maximum ordinate, representing a current of 390 amperes, would be 15½ inches, whilst the ordinate representing the root-mean-square current 7 would only be 1-5 of a millimetre.

You will notice, then, that the fifty groups of the oscillations taking place in one second only occupy in all 1-000 of a second, hence 999 thousandths of a second nothing is happening, and this explains how it is that the

maximum current is so enormously greater than the root-mean-square current.

We must next consider some facts connected with the decay of oscillations in a train. The reason for this decay is evidently the dissipation of energy from some cause or causes. This dissipation may arise from any or all of the following causes :—

1. The resistance of the inductive circuit.
2. The resistance of the spark.
3. Magnetic hysteresis in the metal of the circuit.
4. Di-electric hysteresis or brush discharge or conduction in the condenser.
5. Radiation of energy from the circuit.

This last cause is generally the most important. In closed or nearly closed oscillatory circuits containing a spark gap, the decay of the oscillations is chiefly due to spark resistance, and generally speaking the resistance of the metallic part of a circuit is negligible compared with that of the spark. The decrement per semi-period may be of the order of ·01 or ·001. On the other hand in open oscillatory circuit, such as an Hertzian radiator or Marconi telegraphic aerial, it is found that the decrement per semi-period is much larger and may have the value ·1 or ·2 and this decay is not due to spark resistance, far less resistance of the wire, but chiefly due to the energy dissipated in the form of electric waves.

There are several methods by which this decrement may be determined, and it is essential to distinguish between that part of the decrement which is due to resistance, which is called the resistance decrement and that part due to radiation, which is called the radiation decrement.

Rutherford, Bjerknes and Drude have all devised methods for determining the decrement of electrical oscillations in a circuit. Rutherford's method depends on the principle that if a small bundle of soft iron wires, very fine and short, is magnetised to saturation, and then placed near an oscillatory circuit, it will be demagnetised proportionately to the strength of the first oscillation, which is in the right direction. Since the oscillations alternately magnetise and de-magnetise the steel, and since the sum of a geometrical progression is proportionally to its first term, the integral effect is proportional to the magnitude of the first oscillation. Rutherford, therefore, arranged an apparatus, as follows:—Near the oscillatory circuit he placed a very small bundle of steel wires magnetised to saturation, and thereby caused to deflect a magnetometer. He permitted the discharge to pass first in one direction, and then in another, by charging the condenser alternately in opposite directions, and noticed in each case the loss in magnetism of the small magnet. The ratio of these readings gives us the ratio of the first and second oscillations, and a Napierian logarithm of this ratio gives us the value of the decrement, δ. When there is no radiation, it can be shown that the decrement, δ, is equal to the quotient, $R/2L$, where R is the resistance of the spark and circuit, and if the resistance of the metallic part of the circuit is small, we obtain the value of the spark resistance. Rutherford used this method to measure spark resistance, and also to prove that electrical oscillations are confined to the surface of a conductor. If the metallic part of the oscillatory circuit is made of iron wire, the value of the decrement is largely increased by reason of the hysteresis of the iron, but Rutherford found that the thinnest possible deposit of electrolytic copper put on the iron wire brings the decrement back to the same value as if the wire was solid copper.

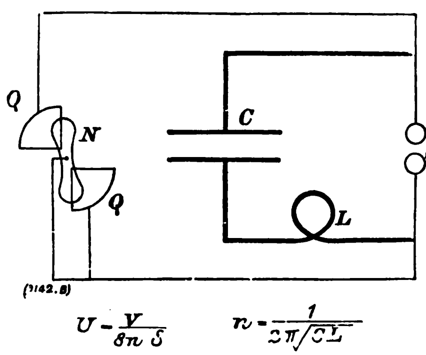
Rutherford's experiments have been more recently repeated by Miss Brooks, and, amongst other things, she has been able to show that the light and the heat of the spark must be chiefly due to the impact of electrons against air molecules. Since the charge consists of a movement of electrons across the gap, and since we know the quantity of electrons moved, we know the number of electrons.

Miss Brooks has given a proof that more electrons are created by the spark than are necessary to carry the current across. This is no doubt due to the ionisation of the air molecules by electronic impact, and it is to the recombination of these ions that the heat and light of the spark are due.

The second method of obtaining the decrement is due to the Norwegian physicist Bjerknes. If we connect to the terminals of a condenser an electrostatic voltmeter, the readings of this voltmeter give us the root-mean-square value of the alternating voltage. (See Fig. 24.) If we measure the spark length, and determine from that the maximum voltage to which the condenser is charged, we can determine the ratio of this last quantity V to the root-mean-square value of the potential, U, and it can easily be shown that the ratio $V/U = 4\alpha = 2R/L$. Hence from this ratio and the known value of the

inductance of the circuit the spark resistance can be determined. A third method, due to both Bjerknes and Drude, is based upon the use of what is called a resonance curve. (See Fig. 25.) It will be necessary to defer until the next lecture a full consideration of the nature and mode of delineation of such a curve, but

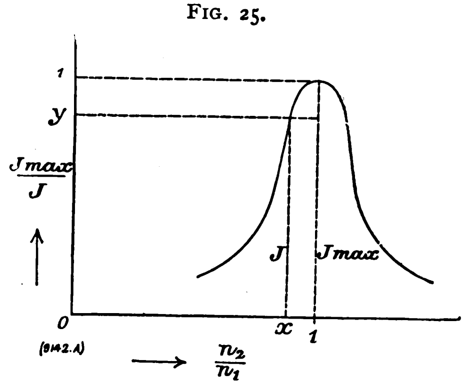
FIG. 24.



BJERKNES' USE OF QUADRANT ELECTROMETER TO MEASURE R.M.S. VALUE OF HIGH FREQUENCY VOLTAGE.

it affords a means of determining with great accuracy the decrement of a circuit, and hence the spark resistance if that circuit contains the spark gap. By the employment of this method Drude showed that for every condenser circuit containing a spark gap there

FIG. 25.



RESONANCE CURVE.

is a certain length of spark which gives minimum of damping, and also that zinc spark balls give less damping than iron or brass.

Another very important matter is the damping of a Hertzian radiator or Marconi aerial wire by radiation. Hertz gave a well-known formula by which this radiation decrement can be determined, and this can be applied to a wireless telegraph antenna of height h, and diameter d. The proof of this formula is long and difficult. It is as follows :—

$$\delta_r = \frac{\pi^3}{24 \log_\epsilon 2\,h/d} = \frac{0{\cdot}54}{\log_{10} 4\,h/d}$$

Thus, if the height of an aerial is 50 metres, and diameter $\frac{1}{10}$th of an inch, we find that the decrement has a value 0·12. This implies that there cannot be much more than a dozen complete oscillations in the case of the ordinary form of the plain aerial.

Another important practical measurement is the determination of the ratio of the energy dissipated in the spark to the energy radiated. When a plain Marconi aerial wire, having a spark gap at the bottom is charged and discharged, part of the energy is radiated as electric waves and part is expended in the spark. The efficiency of the antenna as radiator may be expressed as a percentage of the total energy. It can be shown that the energy radiated is to the energy expended in the spark in the ratio of the radiation decrement to the resistance decrement.

Taking a spark gap, say, of five millimetres in length for an ordinary plain aerial, the ratio of the radiation to the resistance decrement is about 10 to 1, and hence the radiation efficiency of the antenna is about 90 per cent. Supposing such an antenna has the capacity, say, of $\frac{1}{5000}$th of a microfarad and is charged to 15,000 volts, corresponding to a five millimetre spark gap 50 times per second, the power given to the aerial is only 1·25 watts and 90 per cent., and this is about 1 watt. Such an aerial can affect a suitable receiver 100 miles away. On the other hand, to supply such an aerial would involve the use of an induction coil taking, say, 10 amperes at 16 volts, and therefore, although the efficiency of the antenna *per se* is comparatively high, there is a considerable loss of energy somewhere else. This loss of energy takes place in the induction coil, and in the arc discharge which accompanies the spark of the induction coil, hence the moral is that the improvement of the efficiency of a wireless telegraph plant is to be looked for in an improvement in the efficiency of a spark-producing device.

Lecture III.—Delivered December 11, 1905.

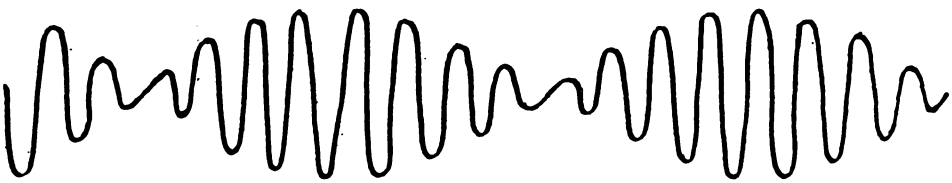
Measurement of Frequency and Resonance.

We have seen in previous lectures that every electrical circuit containing a condenser and inductance when charged and discharged executes electrical oscillations with a certain definite period depending on the capacity and inductance. The circuit may, therefore, be compared to a pendulum which has a periodic time depending on its length. Suppose that we have two such oscillatory circuits placed near to each other in the right position, oscillations set up in one circuit will induce oscillations in the other circuit, and the circuits are said to be inductively coupled. They exercise a mutual action and reaction, and may be compared to two pendulums coupled together mechanically. Thus, for instance, suppose we have two pendulums of equal length hanging from a loose cross string, and that we set one pendulum swinging, it imparts little jerks to the cross string, and if the other pendulum has the same time period, this last will gradually be set in motion. But since action and re-action are equal and opposite, the driving pendulum must retard itself just as much as it accelerates its colleague, and therefore the energy is passed backwards and forwards, hence the pendulum that was started in motion is gradually brought to rest and the other one that was at rest is gradually set in motion. These states of rest and motion alternate for each pendulum. Two such coupled pendulums are called syntonic pendulums when they have the same time period or when one pendulum has a period which is the harmonic of the other. The same experiment may be shown with two pendulums, one of which is double the length of the other. It will not succeed, however, unless the natural time periods of the two pendulums are either equal or in some integer ratio. Similarly two electric circuits when inductively coupled act and react on each other if their time periods are equal, or in integer ratio.

Returning to the pendulums, let us suppose that we take a trace of the motion of the bob of each pendulum, by causing it to mark its vibrations on a strip of paper which is moved uniformly across its motion. We should obtain a curve for each, similar to that in Fig. 26, waxing and waning as the amplitude of the pendulum motion is increased, and decreased.

It is a well-known fact that such a curve as that shown in Fig. 26, is the resultant of two simple harmonic motions of slightly different period, and it is the graphic representation of the audible effect called beats in music, due to the simultaneous action of two sources of sound, say two organ pipes slightly out of tune with each other. Hence we may resolve the waxing and waning motion of each of the coupled pendulums into the sum of two simple harmonic motions of different period. Therefore the effect of coupling two pendulums of equal time period is to generate in each of them by their mutual re-action, a compound oscillation, composed of two vibrations of different periods. The same thing holds good in the case of two electric oscillatory circuits or a pair of coupled circuits constituting an oscillation transformer. These are said to be closely or loosely coupled according as their mutual induction is large or small, the coefficient of coupling denoted by the letter k being equal to $M / \sqrt{LN}$, where M is the mutual inductance of the two circuits, and L and N are the inductances of the two separate circuits.

Fig. 26.

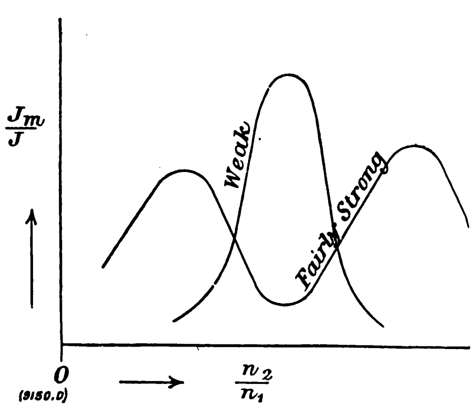


Curve Compounded of Two simple Periodic Curves of slightly different Wave Lengths.

Suppose then we have two such circuits inductively coupled, and electric oscillations are set up in one of these circuits by introducing a spark gap into it and charging the primary condenser by means of an induction coil. We may so adjust the inductance and capacity of the other circuit that it has the same time period, and the circuits are then said to be syntonised or tuned.

If we introduce into the secondary circuit a hot wire ammeter, and measure the effective value of the current in the secondary circuit, we find that in proportion as the secondary circuit is brought into tune with the primary circuit, so does this secondary current increase up to a maximum value which is called the resonance value. We can plot a curve called a resonance curve, ordinates of which represent the ratio of the maximum or resonance current to any other current and corresponding abscissæ represent the ratio of the natural frequencies of the two circuits (see Fig. 27).

FIG. 27.



RESONANCE CURVES.

The form of this resonance curve will depend upon the nature of the coupling. If the coupling is very weak, then the resonance curve has one maximum value. If the coupling is stronger, then it has two maxima like the hump of a dromedary camel. When the tuning of the two circuits is not quite exact, theory shows that there are two sets of superimposed oscillations in the circuit. The resultant oscillation is the sum of these two. When the two circuits are not strongly coupled and are tuned to the same period n_0, then it can be shown that the frequencies n_1 and n_2 of the two resultant oscillations in the secondary circuit are given by the formulæ—

$$n_1 = n_0 \frac{1}{\sqrt{1-k}}$$

$$n_2 = n_0 \frac{1}{\sqrt{1+k}}$$

where $k = M / \sqrt{LN}$

and accordingly we have

$$n_1^2 + n_2^2 = 2n_0^2$$

$$k = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}$$

To illustrate and investigate these facts I have devised an instrument called the cymometer, which has proved itself to be exceedingly useful in connection with high frequency measurements. The principle on which this cymometer is constructed, is as follows :—

It consists (see Fig. 28) of a sliding tubular condenser, formed of two brass tubes separated by an ebonite tube. The outer tube can be moved by a handle, *h*, and an index pointer, P, moves with it over a divided scale, SS. Parallel with the condenser is an inductance coil, H, H, consisting of bare copper wire, wound on an ebonite tube. From the outer tube of the condenser, O, a pin, *l*, having a crutch end, lies on the inductance, and the circuit is completed by a copper bar L_2, L_3, of square section. With the instrument is supplied a vacuum tube, V, which is attached to two small hooks placed on the ends of copper wires, which are respectively in connection with the outer and inner tubes of the condenser. The special characteristic of this instrument is that its inductance and capacity are varied simultaneously and in the same proportion by one movement of a handle.

One vacuum tube consisting of a tube of uranium glass filled with rarefied carbonic dioxide is supplied with the instrument, but by special arrangement a tube filled with the rare gas Neon, can be supplied. The Neon vacuum tube, as I have shown, is far more sensitive than any other form of vacuum tube, and its glow is so brilliant that it can be seen in a large room in broad daylight.

The instrument is employed in the following manner :—

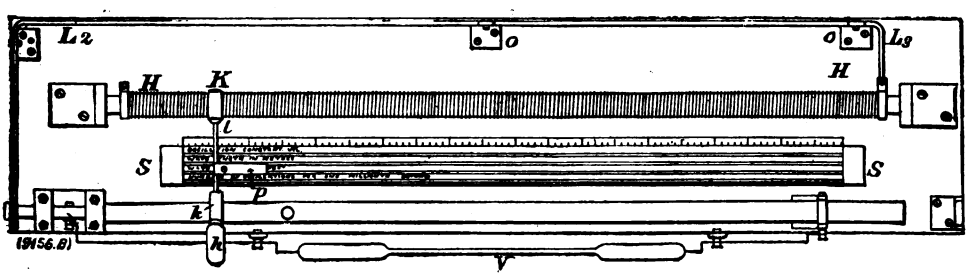
1. *To Determine the Frequency of the Electric Oscillations in any Circuit.*—Place the cymometer so that the copper bar L_2 L_3 is parallel with, and close to any straight portion of the circuit in which electric oscillations are taking place. Then fix the vacuum tube to the two small hooks in connection with the terminals connected to the inner and outer brass tubes forming the condenser, and screw the ebonite handle into the thick collar, *k*, of the outer

tube of the sliding condenser. Move the handle, thus sliding the outer tube of the condenser along, until the vacuum tube glows most brightly. Then the end of the index, P, will indicate on the lowest of the four scales the number of oscillations in one-millionth of a second. Thus, suppose it reads 3·5. This indicates that the frequency of the oscillations is 3·5 millions. Also the top scale reading indicates the *oscillation constant* of the circuit being tested, viz.: the square root of the product of the capacity in microfarads and inductance in centimetres of the cymometer circuit in the position in which it is at that instant, and therefore or any other circuit which is loosely coupled and in resonance with it. If then we know either the inductance or the capacity of that circuit, we can determine the second quantity. The range of the oscillation constant for the instrument illustrated is from 0 to 12. The frequency of the oscillations is connected with the oscillation constant by the rule, Frequency = $5{\cdot}033 \times 10^6$ divided by the oscillation constant. Hence the product of the oscillation constant as recorded on the scale, and the number of oscillations in one millionth of a second is almost exactly equal to the number 5.

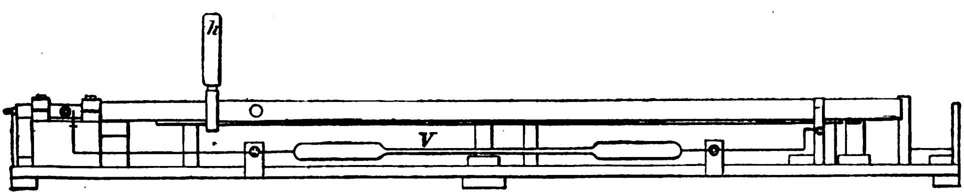
2. *To Determine the Capacity of a Leyden Jar.*—In the lid of the box containing the cymometer is provided a rectangular circuit of insulated wire, the inductance of which is 5,000 centimetres. The rectangle has two tails of wire. It is used as follows:—Set up an induction coil and place the secondary spark balls within a few millimetres of each other. Join one pair of ends of the rectangle of wire to these spark balls. At the other end connect the tails of wire to the inner and outer coatings of the jar to be tested. (Insulate the jar.) Then slide the outer tube of the condenser of the cymometer along until the vacuum tube glows most brightly. Observe on the scale the corresponding reading, marked "oscillation constant." Square this number and divide by 5,000, the resulting quotient is the capacity of the Leyden jar in fractions of a microfarad. The size of capacity which can be measured is determined by the range of the cymometer. In the instrument before you that range covers oscillation constants from 1 to 12.

3. *To Determine the Inductance of a Coil of Wire.*—Determine in the above manner the capacity of a small Leyden jar, say, one having a capacity of about $\frac{1}{700}$th of a microfarad. Then insert in the circuit in series with the rectangle of wire, the piece of wire or coil of which the inductance is required. Move the handle of the cymometer until the vacuum tube shines again most brightly. Observe in each case the reading of the oscillation constant on the scale. Square these numbers and take their differences and divide by the capacity of the Leyden jar, reckoned in microfarads. The quotient is the inductance of the piece of wire in centimetres. Thus, suppose the capacity of the Leyden jar

FIG. 28.



FLEMING CYMOMETER (PLAN).



ELEVATION.

to be $\frac{1}{500}$th of a microfarad, and suppose that in the first case the oscillation constant is found to be 2, and in the second case it is found to be 7, the difference of the squares is then 45, and the inductance of the piece of wire would then be 22,500 centimetres. In this manner an inductance up to about 70,000 centimetres or 80,000 centimetres, that is 70 or 80 microhenrys, can be measured.

A variety of other measurements can be made with the cymometer, which will suggest themselves if it is remembered that when the cymometer bar is placed parallel and near to any circuit in which oscillations are taking place, which has, therefore, capacity and inductance, the scale reading when the vacuum tube glows most brightly, taken on the scale marked "oscillation constant," gives the square root of the product of the capacity reckoned in microfarads, and the inductance reckoned in centimetres of the circuit being tested.

The cymometer is most valuable as a teaching instrument to illustrate to a class or audience the laws of electrical resonance and the phenomena connected with the inductive production of electrical oscillations. It is in fact as useful in connection with high frequency oscillations as a Wheatstone's Bridge or Potentiometer is in relation to the measurement of continuous electrical currents.

The cymometer is contained in a strong wood box fitted with handles. In the lid of this box the standard inductance is fixed.

The over-all dimensions of the instrument are: Length, 4 feet 6 inches = 136 cms.; width, 1 foot = 30 cms.; depth (with handle unscrewed), 6 inches = 15 cms.

We may employ this cymometer to prove the statement just made that in the case of two coupled circuits, oscillations of two frequencies are set up if they have the same natural time period. I have before me two rectangular circuits, each of which contains a Leyden jar, the two circuits have the same inductance and the same capacity, one of these circuits contains a spark gap and we set up oscillations in it in the usual manner by an induction coil. When this is done, if we apply the cymometer to both these circuits, putting the bar of the cymometer first parallel to the secondary circuit and then parallel to the primary circuit, we find that in each of these circuits there are two oscillations, and that the vacuum tube of the cymometer illuminates itself for two positions of the handle of the cymometer. We can determine the natural period of vibration of each of the circuits by removing the secondary circuit and employing the cymometer in the above-described manner to measure the oscillations of one single frequency, then occurring in the primary circuit. We then find that the frequency of these two oscillations and also that of the natural oscillation are different, and comply with the formula given above; one of these oscillations in the coupled circuits has a greater value, and the other has a lesser value than the natural frequency of both the circuits taken separately.

We can also employ this cymometer to determine the oscillations in a wireless telegraph antenna, as follows:—

Place the cymometer with its copper bar parallel to the lower portion of the aerial wire and a few inches from it, then set the transmitter coil in action and move the handle of the cymometer until the vacuum tube glows most brightly. The reading on the scale will give the wave length both in feet and metres. It will in general be found that there are two positions of the handle in which the tube glows. These correspond to the two wave lengths sent out from the aerial.

In making this experiment, and in fact in all cases in which the cymometer is used, as above described, the copper bar L_2, L_3, should be placed as far from the circuit being tested as possible. If it is placed too far away however, the vacuum tube will not glow under any circumstances, but if it is just brought near enough, a position will be found with a very sharp scale reading, in which the vacuum tube just glows.

In the previous courses of lectures I have explained the mode in which the oscillations in such an antenna throw off electric waves of definite wave length. The relation between the wave length λ of the radiated wave measured in centimetres, and the frequency n is given by the formula—

$$3 \times 10^{10} = n\lambda$$

Hence, since the frequency itself is expressed by the formula—

$$n = \frac{5 \times 10^6}{\sqrt{CL}} = \frac{5 \times 10^6}{O}$$

where O is the oscillation constant $= \sqrt{CL}$, we can determine the length of the radiated wave when we know the oscillation constant of the antenna. This can be at once determined by placing the cymometer near to it, and moving the handle of the cymometer so as to adjust its capacity and inductance until the Neon tube shines most

brightly. When this is the case, the scale reading of the cymometer shows its oscillation constant and therefore also that of the antenna. The wave length of the radiated wave is therefore given by the following formulæ—

$$\lambda = \frac{3 \times 10^{10}}{n}$$

$$n = \frac{5 \times 10^{6}}{O}$$

$$\therefore \lambda = \frac{3}{5}\, 10^{4}\ O$$

or $\lambda = 6{,}000 \times O$ cms.
$= 60 \times O$ metres
$= 200 \times O$ feet

Accordingly the cymometer enables us to measure the wave length of the radiated wave and hence its name, from the Greek word *κυμα* = a wave.

In those cases in which we are using an antenna coupled to a closed syntonic circuit, waves of two wave lengths will be thrown off from the antenna in accordance with the principles just explained. These waves have wave lengths given by the formula—

$$\lambda_1 = \lambda_0 \sqrt{1 + k}$$

$$\lambda_2 = \lambda_0 \sqrt{1 - k}$$

where $$\lambda_0 = \frac{3 \times 10^{10}}{n_0}$$

n_0 being the natural frequency of the antenna taken alone.

In such a case the cymometer shows us not only that there are waves of two wave lengths thrown off from the antenna, but that these waves are of unequal insensity and damping. The wave which has the greatest wave length has the least damping and the greatest energy. The wave lengths of these two waves are connected with the natural wave length of the antenna taken alone by the formula—

$$\lambda_1^2 + \lambda_2^2 = 2\lambda_0^2$$

and the coefficient of coupling of the oscillation transformer can be shown to be given by the formula*—

$$k = \frac{\lambda_2^2 - \lambda_1^2}{\lambda_2^2 + \lambda_1^2}$$

For certain reasons it can be shown that the best results are obtained in the case of inductively coupled antennæ when the coupling coefficient, k, has such a value that one of these radiated waves has three times the wave length of the other.

* For the proof of this and other formulæ given in this lecture the reader is referred to the author's treatise on "The Principles of Electric Wave Telegraphy" (Longmans and Co.), or to the German work, Dr. J. Zenneck, "Elektromagnetische Schwingungen and Drahtlose Telegraphic."

The damping of the two radiated waves of different wave lengths is as already observed, different, but the decrements of the two waves are related to the decrements of the primary and secondary circuits when taken separately and this relation has been determined by a formula, due to Drude. If δ_1 is the decrement of the primary or condenser circuit with the spark gap when taken alone, and if δ_2 is that of the antenna taken alone, then Drude had established the following relations :—

$$D_1 = \frac{\delta_1 + \delta_2}{2}\,\frac{n_1}{n_0}$$

$$D_2 = \frac{\delta_1 + \delta_2}{2}\,\frac{n_2}{n_0}$$

Where D_1 and D_2 are the decrements of the two waves of wave length, λ_1 and λ_2 thrown off by the antenna.

We see from the above formula that if we make k nearly unity or employ a very strong coupling, we can make one of the waves radiated from the antenna have much less damping than would be the case for a plain antenna alone. This establishes the importance of the inductive coupling of an antenna with a condenser circuit.

We can make use of the cymometer in a very large number of investigations in connection with high frequency currents. In fact, it is to high frequency currents what the direct reading potentiometer is to low frequency currents, a measuring instrument of wide application. For instance, we can use it to show that high frequency oscillations are confined entirely to the surface of conductors. If we construct two circular loops of wire, one of copper and one of iron of exactly the same size, then we find on attempting to measure the inductance of these loops with a cymometer as above described, that the iron loop damps out the oscillations and that it is difficult to obtain a reading with the iron wire. The cymometer has to be brought much closer to the primary circuit to cause the Neon tube to glow. If, however, we employ a galvanised iron wire of the same diameter and length as the same iron wire, or if we deposit electrochemically on the plain iron wire a thin coating of copper, we find that the inductance is the same as that of a copper wire and that the damping of the oscillations as shown by the behaviour of the Neon tube is the same as if the wire were zinc or copper.

We can also use the cymometer most advantageously to compare dielectric constants,

or if we measure the capacity of a plate condenser formed of two thin plates of metal close together in air, and then insert a sheet of glass or ebonite between the two and measure the capacity again, the ratio of these capacities gives us the dielectric constant of the insulator. It is not difficult to show by experiments of this kind, that the dielectric constant of glass becomes reduced if the frequency is increased.

A simple formula for the capacity of an air condenser formed by two parallel plates is as follows:—Capacity in microfarads = area of plate in square centimetres divided by thickness of air space in centimetres multiplied by 11060000.

I have found this formula useful in connection with experiments on high frequency circuits.

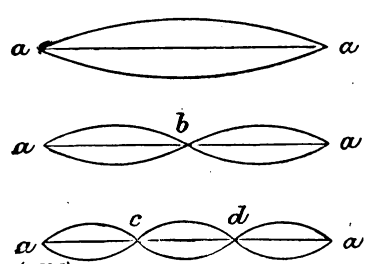
Lecture IV.—Delivered December 18, 1905.

Stationary Electric Waves on Wires.

In acoustics we are not only familiar with progressive waves travelling through space, but also with so-called stationary waves in pipes or on strings. In the case of an organ pipe there is a variation of pressure and velocity from point to point in the pipe, and in the case of vibrating violin strings there are portions of string in strong movement and portions which are nearly at rest. By properly damping the string with the finger the violinist can cause it to sound not only the fundamental note of the string but also a higher harmonic. In this last case the string vibrates in what are called loops and nodes, the node being the place of small movement and the loop the place of greatest amplitude (see Fig. 29). In the same way in an organ pipe there are loops and nodes of pressure and velocity. At the mouth of the pipe the velocity changes are greatest and the pressure changes are nil, but the range of pressure change is greatest. We can also see the same phenomena on the surface of the water. If we create a splash by throwing a stone on to a lake, we set in motion free waves each travelling over the surface, but we can also set up stationary waves in a finger bowl of water by rubbing the finger on the edge or drawing a violin bow over it. In the some way we cause stationary waves of electric potential to be created on wires, or free waves to move along them.

Consider, then, in the first place, a very long wire immersed in an insulator; it has a certain capacity and inductance per unit of length, and also a certain wire resistance and dielectric conductance per unit of length. If at any point of this wire we apply an alternating electromotive force, then at any instant any section or element of the wire will have a different electric potential in its two ends, and less current flowing out of it than flows into it.

Fig. 29.



Violin String vibrating in Fundamental and Harmonics.

The drop in potential is due to resistance and inductance of the wire, and the drop in current down the element is due to its capacity and the conductance of the di-electric. We can easily write down certain mathematical equations which express this fact symbolically. These equations also tell us that the voltage of the current does not instantly appear at all parts of the wire, but that if an alternating electro-

motive force is applied at one place, the potential and current are propagated along the wire with a velocity measured by $1/\sqrt{CL}$, where C and L denote the capacity and inductance per unit of length of the wire. Also the amplitude of the current and potential is weakened as we pass along the wire, in a ratio depending on R and K, the resistance of the wire and the conductance of the dielectric per unit of length of the wire.

I have designed a model to illustrate this gradual diminution in the amplitude of the potential or current taking place in an infinitely long cable when alternating electromotive force is applied at one end. The model consists of a long steel shaft, having on it a number of eccentric wheels, over which pass strings of equal length carrying weights. These eccentric wheels have progressively less eccentricity as they proceed from one end of the rod to the other, and they are also so set on the shaft that each eccentric is in advance of its neighbour by a certain angle. When the shaft carrying the eccentrics is turned round, the weights on the end of the strings rise and fall with a simple periodic motion, but the amplitude of each bob is less than that of the preceding bob, and there is a shift in phase as we pass from bob to bob. Thus you see when the handle is turned a kind of wavy motion passes along the row of bobs, which is a mechanical analogue of the movement of current in a telegraph or telephone cable.

In the next place suppose that the cable is finite in length, the result then is that the wave of potential or current is reflected at the open end, and by interference with the direct wave produces standing waves of potential, just as when a rope fixed at one end is appropriately jerked at the other. In order that these standing waves may be produced, it is necessary, however, that the length of the finite wire shall bear a certain relation to the wave length of the potential wave on the wire. The distance from one loop to the next loop, or one node to the next node, is called half a wave length, and the relation between the wave velocity, wave length and frequency is given by the well-known formula, *wave velocity* = *wave length* × *frequency* Hence, since the wave velocity along the cable is $1/\sqrt{CL}$ if λ denotes the wave length and n denotes the frequency, we must have the following relation :—

$$\frac{1}{\sqrt{CL}} = n\lambda$$

Suppose then that we apply to the end of the cable or wire a high frequency electromotive force with a frequency, n, we can do this by connecting the end of the wire to a condenser circuit having a capacity, C^1, an inductance, L^1, so that the frequency, n, is defined by the equation—

$$n = \frac{1}{2\pi\sqrt{C^1 L^1}}$$

then it follows that the wave length on the wire of which the capacity per unit of length is denoted by C, and the inductance per unit of length by L is given by the equation—

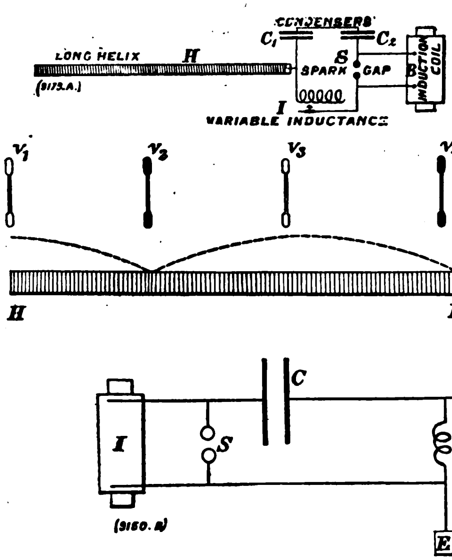
$$\lambda = 2\pi\frac{\sqrt{C^1 L^1}}{\sqrt{CL}}$$

Suppose then that the wire is cut of such a length l, that l bears the following relation to λ—

$$l = \frac{\lambda}{4} = \frac{3\lambda}{4} = \frac{5\lambda}{4} \text{ \&c., \&c.}$$

we shall have stationary waves produced on the wire.

FIG. 30.



STATIONARY WAVES OF ELECTRIC POTENTIAL ESTABLISHED ON A HELIX BY DIRECT COUPLING.

It is very difficult to show this experimeut with a straight wire, because, for any frequency which we can command, we need a wire of such an enormous length, since the potential and current travel along the straight wire with the speed of light, but if we wind a fine, silk-covered copper wire in one layer of closely adjacent turns on an ebonite or glass tube,

we can make a helix or conductor which has large inductance per unit of length, and hence, comparatively speaking, a small velocity of propagation. It is quite easy to make such a helix as that I now exhibit to you, on which the velocity of propagation is only 150th part of the velocity of light. It is for this helix in question 173×10^6 cms., or about 1,200 miles per second. The helix before you has the total length of 210 centimetres and 5,470 turns of wire. The capacity of this helix per unit of length is $C = 45/10^6 \div 210$ microfarads, and the inductance per unit of length, $L = 32 \times 10^6 \div 210$ centimetres. This helix is supported on insulating stands, and one end of it is connected with an oscillatory circuit consisting of a condenser and variable inductance as shown in the diagram, Fig. 30. By properly adjusting this last-named capacity and inductance, we can create a wave of such length on the helix that the length of the helix is about one quarter of the wave length, and we shall then find that we have a stationary wave produced upon the helix which has a node of potential at the generating end, and a loop or maximum of potential at the far end, and a loop or maximum of potential at the far end.

This variation of potential is best shown by means of a Neon vacuum tube which is held near the helix and moved about in the space around. It shows us the distribution of potential by the variation in its glow. We can then vary the frequency of the oscillations in the condenser circuit, so as to make the corresponding wave length on the helix respectively equal to $4\,l/3$, $4\,l/5$, $4\,l/7$, &c., and under these circumstances we find that there are nodes and loops of potential created upon the helix. When the Neon tube is held near a node it does not glow, but when it is held near a loop of potential it glows brightly and we can, as you see, create on the helix stationary electric waves having one loop of potential, two loops, three loops, &c.

The relation between the capacity and the condenser circuit, the inductance of the condenser circuit, and the observed wave length in centimetres on this helix is given in the following Table :—

TABLE V.

OBSERVATIONS MADE WITH A HELIX OF WIRE WOUND ON AN EBONITE CORE, AND WITH AN EBONITE CONDENSER, VARIABLE INDUCTANCE, AND NEON VACUUM-TUBE TO DETERMINE THE WAVE-VELOCITY ALONG THE HELIX.

Oscillation.	Capacity in mfds. in condenser circuit. C.	Inductance in cms. in condenser circuit. L.	Calculated frequency. n.	Observed wave-length in cms. λ	Calculated wave-velocity. $W = n\lambda$ in cms./sec.
Fundamental	0·005835	110,000	$0·197 \times 10^6$	(871)	(172×10^6)
1st Harmonic	0·002887	35,000	0·588 ,,	292	172 ,,
2nd ,, ...	0·001461	18,000	0·977 ,,	175	172 ,,
3rd ,, ...	0·001464	9,000	1·379 ,,	124	171 ,,
4th ,, ...	0·001461	6,000	1·70 ,,	95	163 ,,
5th ,, ...	0·001461	5,000	1·9 ,,	80	152 ,,

From the above Table it is seen that the wave-lengths of the 1st, 2nd, and 3rd harmonics are very nearly in the ratio of 3 : 5 : 7, for $292 \times 3 = 876$, $175 \times 5 = 875$, and $124 \times 7 = 868$. Hence the fundamental wave-length should be 871. If we insert the value in Table V., and calculate the wave-velocity for the fundamental frequency, we find it to be 172×10^6 cms. per second, or exactly the same value as that obtained for the three succeeding harmonics.

Hence, from the fundamental and the three succeeding higher harmonics, we obtain values for the wave-velocity which are closely identical, and equal to 172×10^6 cms./sec., and in very near agreement with the velocity of 174×10^6 cms./sec. calculated from the constants of the helix itself.

There is a slight falling off in the velocity obtained from the 4th and 5th harmonics, but the difficulty of measuring the small inductance then used in the condenser circuit is considerable.

Hence we may say that when a helix wound on an ebonite core is used, observations confirm theory substantially, and show us that the

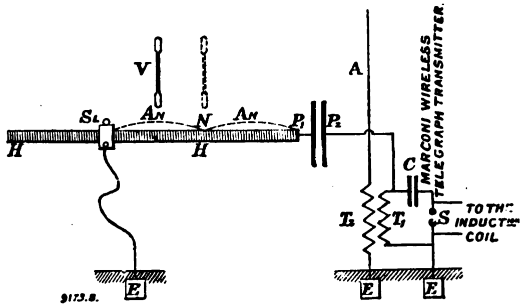
stationary waves are formed as predicted, the wave-velocity being calculable from the observed inductance and capacity of the helix.

This experiment shows us therefore in a very striking manner the production of loops and nodes of potential or current on a conductor. An interesting point to notice is this, that the current in this long wire is not in the same direction at all parts of the wire at the same time, but there are places on this continuous wire in which there is practically no electric current, and other places not far apart in which the current is moving in opposite directions at the same time.

I have applied such a helix as a means of measuring the frequency in a wireless telegraph antenna. Suppose that we have a wireless telegraph transmitter and we desire to know the wave length radiated from the antenna, we can do it in the following manner. Let a long helix made as described be connected through a small condenser with the antenna and let a metal saddle slide on the helix, the said metal saddle being connected by a flexible wire with the earth. (See Fig. 31.) We can then adjust the position of this saddle so that by means of a Neon tube we have one complete stationary wave as indicated by the dotted line set upon the helix. We can measure the length of this wave and knowing the velocity of propagation along the helix we can calculate the frequency of the oscillations. But this frequency must be the same as that of the oscillations in the antenna, and since the wave length of the wave radiated from the antenna is numerically equal to the quotient of the velocity of light, divided by the frequency, we obtain in this manner the measure of the wave length of the radiated wave. Professor Slaby has employed a helix in the same way as an indicator or measurer of the frequency in a wireless telegraph antenna, only instead of using a metal saddle, he shortens or lengthens the helix, more or less, by holding it in the hand, and instead of using a Neon tube he employs some fluorescent material, having particles of gold leaf disseminated through it, spread on paper fixed near the open end of the helix. When the helix is adjusted to be of such a length that this fluorescent paper glows most brightly, then it is known that the length of the helix is approximately one quarter of the stationary wave upon it.

It will be seen, therefore, that there are two classes of cymometers or wave metres for measuring long electric waves. First, the open circuit, or helix type, and second, the closed circuit. One of my own cymometers which I have just described to you and also that of Prof. Slaby, is of the open circuit type, and my other form of cymometer and that of Dönitz, are of the closed circuit type. The characteristic quality of my own form of closed circuit cymometer is that in it the inductance and capacity are varied simultaneously, and in the same proportion by one movement of a handle, and that, therefore, an index can indicate directly on a scale the wave length of the radiated wave, and also render the scale showing the oscillation constant equidivisional. In using any form of cymometer it is essential that we should not alter the quantity we are trying to measure, thus, in using a closed circuit cymometer which is coupled inductively to an antenna, the coupling must be very loose, that is to say, the mutual inductance between the antenna and the cymometer must be small, or else each oscillation in the antenna will give rise to two oscillations of different period in the cymometer, neither of which is that of the antenna, which we are desirous of measuring. Accordingly in my form of cymometer I have so arranged matters that we can get an indication by means of the Neon tube with a very weak coupling indeed between the cymometer and the circuit tested, so that what the cymometer measures is the frequency of the oscillation or oscillations which only exist in the antenna when the cymometer is not there.

FIG. 31.



USE OF A LONG HELIX AS A CYMOMETER FOR MEASURING THE WAVE LENGTH OF A WIRELESS TELEGRAPH TRANSMITTER.

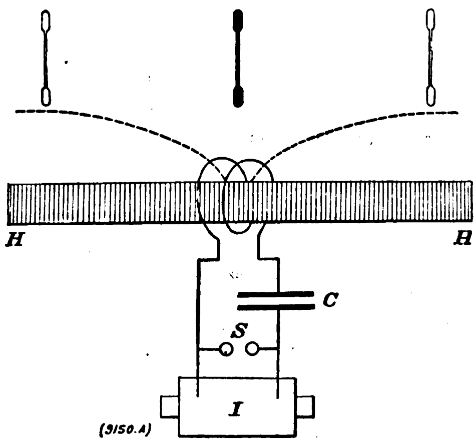
Similarly, in using an open circuit cymometer we have to avoid producing a change in the capacity of the aerial by an earthed conductor held near it, and so alter its natural time period.

It is easy to avoid the above-mentioned

errors with a closed circuit cymometer when using a sensitive detector such as the Neon tube I employ with my form of cymometer. We can also make use of the cymometer shown to you to measure the wave length of the arriving wave as received on the receiving antenna if we substitute a magnetic detector for the Neon tube and coil a few turns of the receiving antenna round the spiral inductance of the cymometer.

If we return again for a moment to the subject of stationary electric waves on spiral wires, I may draw your attention to another method of setting up electrical oscillations on a spiral wire by coupling it inductively instead of directly with the condenser circuit, as shown in Fig. 32. In this manner we can start the free oscillations in a perfectly insulated helix. If

FIG. 32.



STATIONARY WAVE OF ELECTRICAL POTENTIAL ESTABLISHED ON A HELIX BY INDUCTIVE COUPLING.

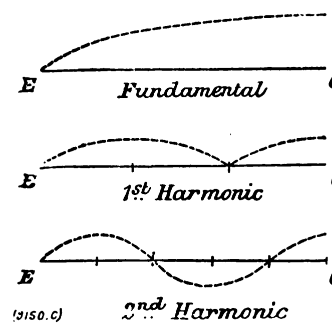
we investigate the distribution of potential of current by means of the Neon tube or a hot wire ammeter, we find that in this case both the free ends of the helix are nodes of current but antinodes or loops of potential, and that the centre of the helix, where the inductive coupling takes place, is a place of current antinode and potential node. By appropriately adjusting the frequency of the condenser circuit we can establish harmonic oscillations on such a helix, and make loops and nodes of potential all along it just as in the case of the spiral insulated at one end only.

In this manner of operating the length of the helix proves to be rather less than half the wave-length on the helix.

It has generally been assumed, and indeed may be shown by a certain simple mathematical analysis, that when oscillations are set up in an insulated linear conductor of the Hertzian type the wave length corresponding to the fundamental oscillation is twice the length of the wire, but it has been shown mathematically by Mr. H. M. Macdonald in his book on "Electric Waves," that the length of waves is really 2·53 times the length of the linear oscillator, and this fact has been experimentally confirmed by Professor Pollock and others.

In experiments with helices I have always found that the distance from the open end of the helix to the first potential node is much less than a quarter of the wave length on the helix, that is to say, is less than half the distance between the two next nodes. Thus reverting to the helix of which data has been given to you, the diagram in Fig. 33 shows

FIG. 33.



STATIONARY WAVES OF POTENTIAL ON A WIRE.

the distribution of the nodes of potential on the helix for the different harmonics used, and we see that the distance from the end of the helix to the first node is always less than half the distance between the two nodes.

Theory shows that the distance between the first and second nodes is always exactly or very nearly half the wave length, hence it follows that when we set up the fundamental oscillation on a helix by coupling it directly at one end to an oscillation circuit, the length of the stationary wave established on the helix is more than four times the length of the helix.

In the same way, when a wireless telegraph antenna of the simple Marconi type is in operation, the wave sent off from it is more than four times the length of the wire, and by experiments with the cymometer I have found it is nearly five times the length of the wire, thus supporting the conclusions derived from

Macdonald's theory. If, however, we insert in the base of the antenna an inductance coil forming the secondary circuit of a transformer, then the conditions are totally different, and it is a more difficult matter to ascertain the length of the wave radiated by such an aerial.

The circuit of the oscillation transformer adds considerably to the effective inductance of the antenna, and we cannot even say that the radiated wave length is five times, far less four times, that of the total length of the antenna and of the oscillation coil taken together.

From experiments made with the cymometer I have ascertained that when using the ordinary forms of oscillation transformer used in wireless telegraphy the relation between the length of the wave radiated from the antenna to the length, l, of the wire forming the antenna itself and the length, l', of the wire forming the secondary circuit of the oscillation transformer in series with the antenna, is given by an empirical formula of the form

$$\lambda = 5(l + Ml')$$

where M is some number lying between 2 and 5.

The only satisfactory method, however, of measuring the length of the radiated wave is to measure as described above, the frequency of the oscillation in the antenna by means of the cymometer. This of course will tell us where there are oscillations of one or two frequencies in the antenna, and will give us the corresponding wave length for these two oscillations.

In practical wireless telegraphy when using oscillation transformers with a coupling coefficient lying between 0·4 and 0·6, one radiated wave may have from 20 to 50 per cent. greater wave length than the other, and the lengths of these radiated waves are connected with the natural free frequency n of the antenna by the formula.

Thus, for instance, at University College, London, I have an antenna consisting of four aluminium wires arranged fan-shaped, each 50′ 6″ in length connected through a semi-rectangle of wire to one end of the secondary circuit of an oscillation transformer the other terminal of which is connected to the earth. The capacity in the primary circuit is 1/40 of a microfarad, and the oscillation constant of this condenser circuit, and the antenna, each taken separately, is 6·9, but when the antenna and condenser circuits are coupled together with a transformer having a co-efficient of coupling of 0·5, I find two resultant oscillation constants in the aerial having values 5 and 8·5 corresponding with waves of 1,000 and 1,700 feet long respectively. The longer of these two waves is the more energetic and the less damped, the ratio of the damping being about 3 to 1.

In conclusion, I may indicate a few outstanding problems in connection with the measurement of long electric waves which have not yet recived complete solution. We have much need of a more simple quantitative wave detector which will provide means for determining the maximum intensity and damping of the waves impinging on a receiving aerial. If we possessed this, and had details as to the sending apparatus used, we should have data which would enable us to demermine approximately the distance of the sending stations, provided we were not working over distances at which atmospheric ionisation becomes objectionable.

In the matter of transmitters, the objections to the use of a spark gap are many and serious. It will no doubt be found possible at some time to solve the important practical problem of producing continuous trains of electric waves of known or regulated amplitude. When this is done many other problems will receive solution, but in spite of numerous patents and announcements nothing satisfactory has yet been found. The important practical problem of locating the radiant point has attracted many inventors, and it is possible that before long a solution may be arrived at in a manner least expected.

Such progress has, however, been made in dealing quantitatively with the problems of high frequency currents and electric waves that we have every confidence that in time these outstanding problems will be successfully solved.

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